Problem **16**

In this problem, we have a block B, that sliding along a slippery icy surface that has no friction. And it hits this slender rod of mass four kilograms in length five meters with an initial speed of 14 meters per second. If the angular velocity of the bar just after the impact is 2.85 radians per second, what is the coefficient of restitution between the slender bar and the block. So what we have to do in this problem is we have to determine this coefficient of restitution for that impact between block and the bar, that results in the bar having an angular velocity of 2.85 radians per second after the right after the impact. So we're going to apply conservation of momentum. And what we're gonna do is we're going to do everything in terms of, which is the top point over here. And this is the point about which this slender rod is rotating. And so that, that'll be our reference. So what we're going to do is we're going to set h o of one being equal to H O of two. So one we define as right before impact. So that's when this block is right here. But it's hasn't contacted yet right before impact. So at this point, we have the block having a velocity and the cylinder bar being stationary, no rotation, no, no velocities, right. Whereas in state two, this is right after the impact, we have both the block and the bar having a velocity, right, that's right after the impact, and the impact is presented by the coefficient of restitution, II, which is what we're tasked to find. So we now have to come up with expressions for each right. And again, this would be the vectorial form, we're going to reduce it down and solve it into a scalar equations. But for now, we're going to keep the vector equations. So H O one is going to be equal to m b times r of C with respect to O cross the B. And this is VB one, right? Why is this? Well, this is because at one, we nothing, we don't care about this, this bar, right? Because this bar is stationary is not moving, so we only care about B. And what we're saying here is that about oh, we have this block with a velocity, right. So M be the mass v, r c with a specto cross v be one, that's this radius down that way, crossed to the velocity of at this point, but the velocity of B, right, which is the initial velocity of the block, which are given in the problem is 14 meters per second. Right? Now, we know that these two are perpendicular because our c with respect to O points downwards, and then VB points to the right, so we're gonna get a vector that actually points out of the page perpendicular to it. And that's how we're going to get rid of essentially the, the vectors, right? Let's look at two. So H O two, this is going to be equal is going to have two components, right, so it's going to have one component due to the velocity of this block over here. So the momentum of this block over here, and then one due to the slender rod, right, so let's start with B. So MB times again, this is going to be RFC with respect to O cross to V B, but this time VB to write the velocity after impact, which we're also given. Plus, we're going to have the component due to that cylinder rod. So, this is going to be I naught times omega A two right and this is I so this is we can just replace this four cylinder rod pinned about oh, this is 1/3 l squared, which is also equal to 1/3 and a L squared, right. And omega here, this we can also represent as V c divided by All right, because V equals two omega cross r. Since omega, in this case, we're gonna go in this direction, we know that so out of the page, R is going to be in the downwards direction as follows, then we know that velocity is going to point to the right. And so the vector form of VC will omega will be VC over L in the k hat direction, right. So that's going to be the vector for omega two, omega A two. And this is supposed to be VC two, right? Because it's at the second state, there is no oh my god at the initial state at the initial one, right. So we've now come up with the dissipation, we're going to combine everything together and put everything into one main equation. So we have m b times r of C with respect to O cross v be one is equal to m b times r of C with respect to O cross v v two plus I naught, which is 1/3. And a L squared times the c two over L. And then this is going to be in the k hat direction. Alright, we can now see, we now know that given that r of C with respect to O is equal to negative L in the j hat direction, you can solve for these and that V is always going to be in the so going back to the diagram here, we know that our Pseudospectral is in the negative j hat direction points down that lane, it has magnitude of L, right? Because it's the length, we know that the velocity is always going to be in this direction over here, right? To the right, so that is in the positive. I had direction. So we had the unit vector in the direction v is equal to i hat, we know that these cross products will yield everything in the positive k hat direction. So we can actually simplify this, these cross products and have them as direct multiplications. Because the vectors are perpendicular, and the resulting vector is gonna be in the k hat direction. So we have m b times L, the B one is equal to MB times L, the b two, and 1/3. And a L squared times V c two over L. So in reality, this simplifies to just L. And everything is going to be in the k hat direction. Right? So we now have an equation that links so we know all the masses, we know L. So we linked these velocities right with v v one, v v two and V c two, right? We said that VC one equals to zero. That's our other equation, right? Because the bar c does not rotate initially. So at one, there is no velocity at sea, before impact. But now we need to find some more equations, right? And again, due to that impact, we can use the we need to find the coefficient of restitution. So we can use the relation between the velocities and the coefficient of restitution. Right, so. coefficient of restitution, you know that II coefficient of restitution is going to be equal to V c two minus V b two divided by v b one minus V c one and we just said that V c one is equal to c zero, because there is no velocity initially. So we can actually simplify this expression to the following. The B two is equal to the C two minus the V one times e. Alright, so now we have an expression for VB two, in terms of VC two and VB one, right? And we can use this, we can take this expression for VB two and plug it back into this equation up here. Right? Because we're given VB one we're also given. So if you go back to the question, are given the initial velocity, 14 meters per second. And we're also given the velocity of the bar, right? Right after impact. So that's V c two, right? We're given VC two, we're given VB one we can find, we need to we don't know VB two, right. So we can replace VB two in these equations with things that we know. And the coefficient of restitution, which is what we're trying to solve for. So we introduced this into this equation, so we can actually solve for it, right. And we can do that. And everything simplifies to the following equation. MB BL b, b one equals to m b times L times V c two minus V b one times e plus L over three, and a nice C two. Now we can further simplify everything to the following. V b one, b times e plus one is equal to V c two and B plus A over three. And from this equation, we can solve for EA, right in terms of VC two, and VB one and n m, m and m. Now the last thing which is a minor detail, V c two, we don't know we're not given VC two, right? We're given the angular velocity of the bar. And from this angular velocity using this equation here, we can find VC two, right? So the C two is equal to omega a two times L. Right? So this is going to be equal to omega a two, which is 2.85 radians per second, times L, which is five meters, and this is going to be equal to 14.28 meters per second. So now we have VC two, we have V V one, we have the masses, and we can solve for E. So using plugging everything into the top equation, so this equation over here, we get the following. six kilograms times five meters, times 14 meters per second, is equal to four kilograms times five meters times V c two, which is what we have just calculated 14.28 meters per second minus V b one, which is 14 meters per second times e plus five meters over three times M A, which is four kilograms, times 14.28 meters per second. And with this So for E E is equal to 0.7 and this is our final answer